

Μαθημα 26

Ντογλας 6ος 95 1.35(δ)

$$\int \frac{x \cos x}{(x \sin x + \cos x)^2} dx \quad \frac{(x \sin x + \cos x)'}{1 \cdot \sin x + x \cos x - \sin x} = \frac{(x \sin x + \cos x)'}{x \cos x} \int \frac{(x \sin x + \cos x)'}{(x \sin x + \cos x)^2} dx$$

$$= - \frac{1}{x \sin x + \cos x} + c$$

$$1.37 (\delta) \quad \int \frac{1}{1 + \sin x + \cos x} dx \quad \begin{aligned} u &= \tan \frac{x}{2} \\ \sin x &= \frac{2u}{1+u^2} \\ \cos x &= \frac{1-u^2}{1+u^2} \\ dx &= \frac{2}{1+u^2} du \end{aligned} \int \frac{1}{1 + \frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}} \frac{2}{1+u^2} du$$

$$= \int \frac{2}{1+u^2+2u+1-u^2} du = \int \frac{2}{1+2u} du = \ln|1+u| + c =$$

$$= \ln \left| 1 + \tan \frac{x}{2} \right| + c$$

$$1.37(\eta) \quad \int \frac{dx}{x \sqrt{(x-a)(b-x)}}$$

$$\text{Παρατηρούμε ότι } (x-a)(b-x) = xb - x^2 - ab + ax =$$

$$= - \left(x^2 - (a+b)x + ab \right) = - \left(x^2 - 2 \frac{a+b}{2} x + \left(\frac{a+b}{2} \right)^2 - \left(\frac{a+b}{2} \right)^2 + ab \right)$$

$$= - \left(\left(x - \frac{a+b}{2} \right)^2 - \frac{(a+b)^2 - 4ab}{4} \right) =$$

$$= - \left(\left(x - \frac{a+b}{2} \right)^2 - \left(\frac{a-b}{2} \right)^2 \right) = \left(\frac{a-b}{2} \right)^2 - \left(x - \frac{a+b}{2} \right)^2$$

$$A_1 - I = \int \frac{dx}{x \sqrt{(x-a)(b-x)}} = \int \frac{dx}{x \sqrt{\left(\frac{a-b}{2}\right)^2 - \left(x - \frac{a+b}{2}\right)^2}}$$

Derajat $x - \frac{a+b}{2} = \frac{a-b}{2} \sin t$

$$\Rightarrow x = \frac{a-b}{2} \sin t + \frac{a+b}{2}$$

$$dx = \frac{a-b}{2} \cos t dt$$

$$I = \int \frac{\frac{a-b}{2} \cos t dt}{\left(\frac{a-b}{2} \sin t + \frac{a+b}{2}\right) \sqrt{\left(\frac{a-b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 \sin^2 t}} dt$$

$$= \int \frac{1}{\frac{a-b}{2} \sin t + \frac{a+b}{2}} dt \quad \begin{matrix} u = \tan(t/2) \\ \sin t = \frac{2u}{1+u^2} \\ dt = \frac{2}{1+u^2} du \end{matrix}$$

$$= \int \frac{1}{\frac{a-b}{2} \frac{2u}{1+u^2} + \frac{a+b}{2}} \frac{2}{1+u^2} du =$$

$$= \int \frac{1}{(a-b)u + (a+b)(1+u^2)} du = \int \frac{1}{(a+b)u^2 + 4au + (a+b)} du$$

Επί κ α = β

$$\frac{1}{a+b} \int \frac{1}{u^2 + 4 \frac{a}{a+b} u + 1} du$$

Διακρίνουσ αφημένους αν έχει ή όχι ρίζες. Π.χ.

αν $\Delta < 0$ τότε $I = \frac{1}{a+b} \int \frac{1}{\left(u + \frac{2a}{a+b}\right)^2 + \left(1 - \frac{4a^2}{(a+b)^2}\right)} du$

Kann Dr. zov/r $y = \frac{u + \frac{2a}{a+b}}{\sqrt{1 - \frac{4a^2}{(a+b)^2}}}$ \hookrightarrow so I abhengen

es $\int \frac{1}{y^2+1} dy = \arctan y + C \dots \dots$ (apitana us astun)

1.38(J) $\int \frac{1+x^2}{1+x^4} dx = \int \frac{1+x^2}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)} dx$

$(1+x^4 = (x^2+\alpha x+\beta)(x^2+\delta x+\epsilon) \Rightarrow \alpha=\sqrt{2} \quad \delta=-\sqrt{2} \quad \beta=\epsilon=1)$

$= \int \frac{Ax+B}{x^2+\sqrt{2}x+1} dx + \int \frac{Cx+D}{x^2-\sqrt{2}x+1} dx$ $\left\{ \begin{array}{l} A=C=0 \\ B=D=1/2 \end{array} \right.$

$= \int \frac{1/2}{x^2+\sqrt{2}x+1} dx + \int \frac{1/2}{x^2-\sqrt{2}x+1} dx =$

$= \frac{1}{2} \int \frac{1}{x^2+2\frac{\sqrt{2}}{2}x + (\frac{\sqrt{2}}{2})^2 - (\frac{\sqrt{2}}{2})^2 + 1} dx + \frac{1}{2} \int \frac{1}{x^2-2\frac{\sqrt{2}}{2}x + (\frac{\sqrt{2}}{2})^2 - (\frac{\sqrt{2}}{2})^2 + 1} dx$

$= \frac{1}{2} \int \frac{1}{(x+\frac{\sqrt{2}}{2})^2 + \frac{1}{2}} dx + \frac{1}{2} \int \frac{1}{(x-\frac{\sqrt{2}}{2})^2 + \frac{1}{2}} dx$

$= \frac{1}{2} \int \frac{1}{\frac{1}{2}((\sqrt{2}x+1)^2+1)} dx + \frac{1}{2} \int \frac{1}{\frac{1}{2}((\sqrt{2}x-1)^2+1)} dx$

$= \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} + \frac{\arctan(\sqrt{2}x-1)}{\sqrt{2}} + C$

$$\text{Evarin} \int \sqrt{\tan x} dx \quad \begin{array}{l} u = \sqrt{\tan x} \\ \arctan(u^2) = x \\ dx = \frac{2u}{1+u^4} du \end{array} \int \frac{2u^2}{1+u^4} du$$

To $(1+u^4)$ παραγοντοποιείται ως εξής: γράφεται

$$1+u^4 = (x^2 + \alpha x + \beta)(x^2 + \gamma x + \delta) \quad \text{και ποικίλουν}$$

$$\alpha = \sqrt{2} \quad \beta = 1 = \delta \quad \gamma = -\sqrt{2}$$

$$A_{72} \int \frac{2u^2}{1+u^4} du = \int \frac{2u^2}{(u^2 + \sqrt{2}u + 1)(u^2 - \sqrt{2}u + 1)} du$$

$$= \int \left(\frac{Au + B}{u^2 + \sqrt{2}u + 1} + \frac{\Gamma u + \Delta}{u^2 - \sqrt{2}u + 1} \right) du \quad \text{και βρίσκουμε}$$

$$\left(A = -\frac{\sqrt{2}}{2} \quad \Gamma = \frac{\sqrt{2}}{2} \quad B = \Delta = 0 \right)$$

$$\int \frac{-\frac{\sqrt{2}}{2}u}{u^2 + \sqrt{2}u + 1} du + \int \frac{\frac{\sqrt{2}}{2}u}{u^2 - \sqrt{2}u + 1} du$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} \int \frac{2u}{u^2 + \sqrt{2}u + 1} du + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \int \frac{2u}{u^2 - \sqrt{2}u + 1} du$$

$$= -\frac{\sqrt{2}}{4} \int \frac{2u + \sqrt{2} - \sqrt{2}}{u^2 + \sqrt{2}u + 1} du + \frac{\sqrt{2}}{4} \int \frac{2u - \sqrt{2} + \sqrt{2}}{u^2 - \sqrt{2}u + 1} du$$

$$= -\frac{\sqrt{2}}{4} \ln |u^2 + \sqrt{2}u + 1| + \frac{\sqrt{2}}{4} \ln |u^2 - \sqrt{2}u + 1|$$

$$-\frac{\sqrt{2}}{4} \int \frac{-\sqrt{2}}{u^2 + \sqrt{2}u + 1} du + \frac{\sqrt{2}}{4} \int \frac{\sqrt{2}}{u^2 - \sqrt{2}u + 1} du$$

Νεγρελίνης σελ 376
24-4 (ii)

$$I = \int \frac{x^4}{1+x^2} \arctan x \, dx$$

Θέλουμε να αναλλογήσουμε από την $\arctan x$. Αυτό γίνεται με πολλαπλασιαστική ολοκλήρωση. Αν x χρειάζομαστε αντιστοίχως τον $\frac{x^4}{1+x^2}$

Την υπολογίζουμε ξεχωριστά: $\int \frac{x^4}{1+x^2} dx$ καλύπτει $\int \frac{x^4-1+1}{1+x^2} dx$
διέσπασε 4

$$= \int \frac{(x^2-1)(x^2+1)+1}{1+x^2} dx = \int \left((x^2-1) + \frac{1}{x^2+1} \right) dx =$$

$$= \frac{x^3}{3} - x + \arctan x + C \quad \text{ή}$$

$$I = \int \left(\frac{x^3}{3} - x + \arctan x \right)' \arctan x \, dx =$$

$$= \left(\frac{x^3}{3} - x + \arctan x \right) \arctan x - \int \left(\frac{x^3}{3} - x + \arctan x \right) \frac{1}{1+x^2} dx$$

Μένει να υπολογίσουμε τα $\int \frac{x^3}{1+x^2} dx$ διαιρέσει ---

$$\int \frac{x}{1+x^2} dx \quad \underline{\underline{u=1+x^2}}$$

και $J = \int \frac{\arctan x}{1+x^2} dx$ ~~απόδειξη~~ $\int (\arctan x)' \arctan x \, dx$

$$= \arctan^2 x - \int \arctan x \frac{1}{1+x^2} dx$$

$$= \arctan^2 x - J \Rightarrow J = \frac{1}{2} \arctan^2 x$$

Άλλος(ο) $I = \int (x^4 \arctan x) (\arctan x)' dx =$

$$= x^4 \arctan^2 x - \int \left(4x^3 \arctan x + \frac{x^4}{1+x^2} \right) \arctan x \, dx =$$

$$= x^4 \arctan^2 x - 4 \int x^3 \arctan^2 x - I = ??$$

πρέπει να υπολογίσει το $\int 4x^3 \arctan^2 x \, dx$