Intersection form and stable norms of Finsler surfaces

Daniel Massart

Samos, June 2013

Abstract

Given an oriented surface M, and two $C1$ closed curves α and β which intersect transversally, we denote $I(\alpha, \beta)$ the algebraic intersection of α and β. Now we endow M with a Finsler metric m, and we denote by $l(.)$ the length of a closed curve with respect to m. We would like to estimate the quantity

$$K(M, m) := \sup_{\alpha,\beta} \frac{I(\alpha, \beta)}{l(\alpha)l(\beta)}.$$

In other words, if we view the length as a cost function, we ask: how much intersection can we get for our money ? In these lectures, we propose to show that

- the intersection form induces a symplectic structure on the first homology $H_1(M, \mathbb{R})$ of M
- the metric m induces a norm on the first homology of M, usually called the stable norm.

The quantity $K(M, m)$ may then be viewed as the norm of the bilinear form $I(., .)$ on $H_1(M, \mathbb{R})$, with respect to the stable norm. Just like systolic geometers ask how much geometric information can be contained in the systole, we would like to know what information can be deduced from the value of $K(M, m)$. We are particularly interested in Riemannian surfaces of constant curvature. We shall discuss the following questions :

- when is the supremum in the definition of $K(M, m)$ a maximum ?
- does $K(M, m)$ have a minimum (resp. maximum) when m ranges over the moduli space ? if so, by which metrics m is it realized ?