

A crash course in Finsler Geometry

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Abstract

A disorienting feature of Finsler Geometry for the beginner is the variety of viewpoints, languages and notation systems, making the subject appearing like an archipelago with a number of disconnected islands. In these introductory lectures we will attempt to give an overview of several classical aspects of Finsler Geometry, hoping to convey a sense of unity.

We plan to cover the following topics:

Minkowski Geometry. Minkowski spaces are the “flat spaces” in Finsler geometry. We will define and characterize them, relate them to convex geometry, and describe their isometry groups.

Finsler Manifolds. Definition, basic examples, some classes of Finsler manifolds. Volumes in Finsler Geometry. Relation with the calculus of variation. Geodesics. Exponential map.

The curvature tensor. The Ehresmann connection. Tensors in Finsler manifolds. The Chern Connection. The curvature tensor and its horizontal-vertical decomposition. The Flag curvature. Jacobi fields. Influence of the Flag curvature on the topology.

Projectively flat manifolds. Projectively flat manifolds. Hilbert problem IV. Beltrami theorem and the curvature of Hilbert geometry.

If time permits we will also cover (parts of) the following topics:

The Binet-Legendre metric and application. To any Finsler metric is associated a natural Riemannian metric called the *Binet-Legendre metric*. This metric can be used to solve some problems in Finsler geometry by techniques of Riemannian geometry. Applications to the group of isometries, conformal transformations and symmetric spaces will be given.

Conformally flat Finsler manifolds. A Finsler analogue of the classic Weyl theorem giving necessary and sufficient for a Riemannian manifold to be conformally flat will be discussed.

This course is intended for PhD students working in geometry. No specific requirement is necessary, but some familiarity with Riemannian geometry will be a plus.