Summer School

”Finsler Geometry with applications to low-dimensional geometry and topology”

Program

Monday 03 June 2013

08:30-09:00 *Registration*

09:00-09:50 *Riemann surfaces*
Lecture I **A’Campo**

Coffee Break

10:10-11:00 *The structure of fundamental groups of manifolds with curvature bounded below, after V. Kapovitch and B. Wilking,*
Lecture I **Courtois**

11:00-11:50 *Intersection form and stable norms of Finsler surfaces*
Lecture I **Massart**

Lunch Break

14:10-15:00 *Introduction to systolic geometry*
Lecture I **Babenko**

15:00-15:50 *Sub-Finsler metrics in geometric group theory*
Seminar Talk I **M. Duchin**

Tuesday 04 June 2013

09:00-09:50 *A crash course in Finsler Geometry*
LECTURE I **TROYANOV**

Coffee Break

10:10-11:00 *Riemann surfaces*
LECTURE II **A'CAMPO**

11:00-11:50 *The structure of fundamental groups of manifolds with curvature bounded below, after V. Kapovitch and B. Wilking,*
LECTURE II **COURTOIS**

Lunch Break

14:10-15:00 *Intersection form and stable norms of Finsler surfaces*
LECTURE II **MASSART**

15:00-15:50 *Chark Groupoids and Thompsons Groups*
SEMINAR TALK II **M. ULUDAG**

Wednesday 05 June 2013

09:00-09:50 *Introduction to systolic geometry*
LECTURE II **BABENKO**

Coffee Break

10:10-11:00 *A crash course in Finsler Geometry*
LECTURE II **TROYANOV**

11:00-11:50 *Riemann surfaces*
LECTURE III **A'CAMPO**

Lunch Break

14:10-15:00 *The structure of fundamental groups of manifolds with curvature bounded below, after V. Kapovitch and B. Wilking,*
LECTURE III **COURTOIS**

15:00-15:50 *Triangulations of Sphere After Thurston*
SEMINAR TALK III **I. SAGLAM**

Thursday 06 June 2013

09:00-09:50 *Intersection form and stable norms of Finsler surfaces*
LECTURE III **MASSART**

Coffee Break

10:10-11:00 *Introduction to systolic geometry*
LECTURE III **BABENKO**

11:00-11:50 *A crash course in Finsler Geometry*
LECTURE III **TROYANOV**

Lunch Break

14:10-15:00 *Riemann surfaces*
LECTURE IV **A'CAMPO**

15:00-15:50 *The structure of fundamental groups of manifolds with curvature bounded below, after V. Kapovitch and B. Wilking,*
LECTURE IV **COURTOIS**

20:00 Dinner at the village Kontakeika

Friday 07 June 2013

09:00-09:50 *Intersection form and stable norms of Finsler surfaces*
LECTURE IV **MASSART**

Coffee Break

10:10-11:00 *Introduction to systolic geometry*
LECTURE IV **BABENKO**

11:00-11:50 *A crash course in Finsler Geometry*
LECTURE IV **TROYANOV**

Lunch Break

14:10-15:00 *The structure of fundamental groups of manifolds with curvature bounded below, after V. Kapovitch and B. Wilking,*
LECTURE V **COURTOIS**

15:00-15:50 *Seminar Talk IV*

Saturday 08 June 2013

09:00-09:50 *Riemann surfaces*
LECTURE V **A'CAMPO**

Coffee Break

10:10-11:00 *Intersection form and stable norms of Finsler surfaces*
LECTURE V **MASSART**

11:00-11:50 *Introduction to systolic geometry*
LECTURE V **BABENKO**

Lunch Break

14:10-15:00 *A crash course in Finsler Geometry*
LECTURE V **TROYANOV**

MARGULIS LEMMA, OLD AND NEW

GILLES COURTOIS

The Bieberbach theorem says that any discrete group of isometries of the Euclidean space contains a finite index subgroup whose elements are translations. In particular, discrete groups of euclidean isometries are virtually abelian.

In the late 70's, G. Margulis gave a non-Euclidean version of this theorem, now called the "Margulis lemma" : there exists a constant $\mu(n)$ such that for any n-dimensional symmetric Riemannian space of non-positive curvature X, any $x \in X$, any discrete group of isometries of X generated by elements g such that $d(x, gx) \leq \mu(n)$ is virtually nilpotent. This statement as well as the Bieberbach theorem rely on the algebraic nature of the isometry group under consideration but M. Gromov extended the Margulis lemma in a purely Riemanniann setting : let X be a simply connected n-dimensional manifold of sectional curvature $-1 \leq K \leq 0$, then any discrete group of isometries generated by elements g such that $d(x, gx) \leq \mu(n)$ is virtually nilpotent.

In the 80's, M. Gromov generalized the Margulis lemma stating that the fundamental group of an "almost flat manifold" is virtually nilpotent : there exists a constant $\epsilon(n)$ such that the fundamental group of any n-dimensional compact riemannian manifold such that the sectional curvature K and the diameter diam satisfy $|K|. \operatorname{diam}^2 \leq \varepsilon(n)$ is virtually nilpotent. The sectional curvature assumption here replaces the homogeneity of the space in the classical Margulis lemma and seems to be crucial. However M. Gromov conjectured that the Margulis lemma should hold under a much weaker curvature assumption namely a lower bound on Ricci curvature. This conjecture has been recently settled by V. Kapovitch and B. Wilking. Their proof relies on the theory of Cheeger and Colding which describes the structure of Gromov-Hausdorff limits of sequences of riemannian manifolds with Ricci curvature bounded below.

The goal of this lecture is to survey some of these results and the involved tools.

Intersection form and stable norms of Finsler surfaces

Daniel Massart

Samos, June 2013

Abstract

Given an oriented surface M, and two $C1$ closed curves α and β which intersect transversally, we denote $I(\alpha, \beta)$ the algebraic intersection of α and β. Now we endow M with a Finsler metric m, and we denote by $l(.)$ the length of a closed curve with respect to m. We would like to estimate the quantity

$$K(M, m) := \sup_{\alpha,\beta} \frac{I(\alpha, \beta)}{l(\alpha)l(\beta)}.$$

In other words, if we view the length as a cost function, we ask: how much intersection can we get for our money ? In these lectures, we propose to show that

- the intersection form induces a symplectic structure on the first homology $H_1(M, \mathbb{R})$ of M
- the metric m induces a norm on the first homology of M, usually called the stable norm.

The quantity $K(M, m)$ may then be viewed as the norm of the bilinear form $I(., .)$ on $H_1(M, \mathbb{R})$, with respect to the stable norm. Just like systolic geometers ask how much geometric information can be contained in the systole, we would like to know what information can be deduced from the value of $K(M, m)$. We are particularly interested in Riemannian surfaces of constant curvature. We shall discuss the following questions :

- when is the supremum in the definition of $K(M, m)$ a maximum ?
- does $K(M, m)$ have a minimum (resp. maximum) when m ranges over the moduli space ? if so, by which metrics m is it realized ?

A crash course in Finsler Geometry

Marc Troyanov (EPFL)

Samos, June 2013

Abstract

A disorienting feature of Finsler Geometry for the beginner is the variety of viewpoints, languages and notation systems, making the subject appearing like an archipelago with a number of disconnected islands. In these introductory lectures we will attempt to give an overview of several classical aspects of Finsler Geometry, hoping to convey a sense of unity.

We plan to cover the following topics:

Minkowski Geometry. Minkowski spaces are the "flat spaces" in Finsler geometry. We will define and characterize them, relate them to convex geometry, and describe their isometry groups.

Finsler Manifolds. Definition, basic examples, some classes of Finsler manifolds. Volumes in Finsler Geometry. Relation with the calculus of variation. Geodesics. Exponential map.

The curvature tensor. The Ehresmann connection. Tensors in Finsler manifolds. The Chern Connection. The curvature tensor and its horizontal-vertical decomposition. The Flag curvature. Jacobi fields. Influence of the Flag curvature on the topology.

Projectively flat manifolds. Projectively flat manifolds. Hilbert problem IV. Beltrami theorem and the curvature of Hilbert geometry.

If time permits we will also cover (parts of) the following topics:

The Binet-Legendre metric and application. To any Finsler metric is associated a natural Riemannian metric called the *Binet-Legendre metric*. This metric can be used to solve some problems in Finsler geometry by techniques of Riemannian geometry. Applications to the group of isometries, conformal transformations and symmetric spaces will be given.

Conformally flat Finsler manifolds. A Finsler analogue of the classic Weyl theorem giving necessary and sufficient for a Riemannian manifold to be conformally flat will be discussed.

This course is intended for PhD students working in geometry. No specific requirement is necessary, but some familiarity with Riemannian geometry will be a plus.

Sub-Finsler metrics in geometric group theory

Moon Duchin*

Samos, June 2013

Abstract

I will discuss the geometry of the Heisenberg groups, both real and discrete. By a theorem of Pansu, any word metric on the integer Heisenberg group has the large-scale structure of a sub-Finsler metric on the ambient Lie group. I will describe some consequences that can be drawn from this for the "internal" geometry of each group.

*joint work with Christopher Mooney

Chark Groupoids and Thompson's Groups

Muhammed Uludag*
Galatasaray University

Samos, June 2013

Abstract

We introduce and study an analogue of Thompson's group T. This group appears as the fundamental group of the so-called Chark groupoid, whose objects are certain infinite ribbon graphs called Charks. These graphs can be naturally identified with the set of narrow ideal classes in real quadratic number fields. They are canonically embedded in conformal annuli, with a unique cycle, with several Farey tree components attached to this cycle. Morphisms of the Chark groupoid are generated by flips. They can be identified with the set of indefinite binary quadratic forms. Objects of this groupoid can be naturally identified with classes of indefinite binary quadratic forms. We aim to show that the group associated to the Chark groupoid is an infinite extension of Thompson's group. Along the way we also study three simpler analogues leading to Thompson's group F and to some finite extensions of T. An open question is: what kind of arithmetic information can one extract out of this group

*Joint work in progress with Ayberk Zeytin

Triangulations of Sphere After Thurston

Ismail Saglam

Samos, June 2013

Abstract

A triangulation of sphere is called non-negativeley curved if each vertex is not incident to more than 6 triangles. In his paper ”Shapes of polyhedra and triangulations of the sphere” Thurston described all non-negatively curved triangulations of sphere as the positive part of a 10 dimensional Eisenstein lattice, with signature $(1, 9)$, divided by its automorphism group.

To do this, Thurston considered triangulations as cone metrics on the sphere. Then he used edge unfolding to develop triangulations to plane. But the same method works for Alexandrov unfolding. This has two advantages: we see how triangulations are obtained easily and construction of degenerate triangulations becomes apparent. That is, by this way we can describe subfamilies of non-negatively curved triangulations.