Summer School

”Finsler Geometry with applications”

Program

Monday 22 September 2014

08:30-09:00 *Registration*

09:00-09:50 *Geometry of Milnor fibers*
Lecture I — **A’Campo**

Coffee Break

10:10-11:00 *Geometric structures on manifolds*
Lecture I — **Goldman**

11:00-11:50 *Anosov Representations*
Lecture I — **Guichard**

Lunch Break

13:40-14:30 *Metric projective geometry*
Lecture I — **Matveev**

14:30-15:20 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
Lecture I — **Schroeder**

15:30-16:20 *Compactifications of locally symmetric spaces of infinite volume*
Seminar — **L. Ji**

Tuesday 23 September 2014

09:00-09:50 *Convex geometries of Teichmüller space: a comparative study*
LECTURE I — **YAMADA**

Coffee Break

10:10-11:00 *Geometry of Milnor fibers*
LECTURE II — **A'CAMPO**

11:00-11:50 *Geometric structures on manifolds*
LECTURE II — **GOLDMAN**

Lunch Break

13:40-14:30 *Anosov Representations*
LECTURE II — **GUICHARD**

14:30-15:20 *Metric projective geometry*
LECTURE II — **MATVEEV**

15:30-16:20 *Compactifications of locally symmetric spaces of infinite volume*
SEMINAR — **L. JI**

16:30-15:20 *Strictly convex functions on complete Finsler manifolds.*
SEMINAR — **B. TIWARI**

Wednesday 24 September 2014

09:00-09:50 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
LECTURE II — **SCHROEDER**

Coffee Break

10:10-11:00 *Convex geometries of Teichmüller space: a comparative study*
LECTURE II — **YAMADA**

11:00-11:50 *Geometric structures on manifolds*
LECTURE III — **GOLDMAN**

Lunch Break

13:40-14:30 *Anosov Representations*
Lecture III — **Guichard**

14:30-15:20 *Thurston's asymmetric metric for unusual surfaces*
Seminar — **D. Alessandrini**

Thursday 25 September 2014

09:00-09:50 *Metric projective geometry*
Lecture III — **Matveev**

Coffee Break

10:10-11:00 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
Lecture III — **Schroeder**

11:00-11:50 *Convex geometries of Teichmüller space: a comparative study*
Lecture III — **Yamada**

Lunch Break

13:40-14:30 *Geometric structures on manifolds*
Lecture IV — **Goldman**

14:30-15:20 *Teichmüller spaces as infinite polyhedra*
Seminar — **Y. Matsumoto**

20:00 Dinner at the village Kontakeika

Friday 26 September 2014

09:00-09:50 *Anosov Representations*
Lecture IV — **Guichard**

Coffee Break

10:10-11:00 *Metric projective geometry*
Lecture IV — **Matveev**

11:00-11:50 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
LECTURE IV **SCHROEDER**

Lunch Break

13:40-14:30 *Convex geometries of Teichmüller space: a comparative study*
LECTURE IV **YAMADA**

14:30-15:20 *Geometric structures on manifolds*
LECTURE V **GOLDMAN**

Saturday 27 September 2014

09:00-09:50 *Anosov Representations*
LECTURE V **GUICHARD**

Coffee Break

10:10-11:00 *Metric projective geometry*
LECTURE V **MATVEEV**

11:00-11:50 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
LECTURE V **SCHROEDER**

Lunch Break

13:40-14:30 *Convex geometries of Teichmüller space: a comparative study*
LECTURE V **YAMADA**

14:30-15:20 *Special Families of Hyperbolic Structures on Infinite Type Surfaces*
SEMINAR **O. EVREN**

Sunday 28 September 2014

09:00-09:50 *Metric Möbius Geometry and Boundaries of neg. curved spaces*
LECTURE VI **SCHROEDER**

Coffee Break

10:10-11:00 *Convex geometries of Teichmüller space: a comparative study*
LECTURE VI **YAMADA**

11:00-11:50 *Anosov Representations*
LECTURE VI **GUICHARD**

12:00-12:50 *Metric projective geometry*
LECTURE VI **MATVEEV**

Monday 29 September 2014

09:00-09:50 *Geometry of Milnor fibers*
LECTURE III **A'CAMPO**

Coffee Break

10:10-11:00 *Geometry of Milnor fibers*
LECTURE IV **A'CAMPO**

11:00-11:50 *String kinematics and integrability in higher dimensions*
SEMINAR **R. RICCA**

Tuesday 30 September 2014

09:00-09:50 *Geometry of Milnor fibers*
LECTURE V **A'CAMPO**

Coffee Break

10:10-11:00 *TBA*
SEMINAR **N. SHOJAEE**

11:00-11:50 *Andreev's theorem on projective Coxeter polyhedra*
SEMINAR **G-S LEE**

Abstracts of Lectures

Geometry of Milnor fibers

Norbert A'Campo (Basel)

It is very amazing to see how efficiently polynomials can produce unexpected objects. Local levels of a single polynomial near an isolated singularity can be studied by the pioneering method of Milnor. Such levels are manifolds with boundary that have a very rich panoply of structures.
Keywords for the talks: contact and symplectic manifolds, monodromy, mapping class group, exotic spheres.

Geometric structures on manifolds

Bill Goldman (Maryland)

Geometry on homogeneous spaces
Basic examples: Euclidean and non-Euclidean geometries, affine and projective geometries, conformal geometry, Lorentzian geometry
Deformation theory: setting up the moduli space and the Ehresmann-Weil-Thurston holonomy theorem
Affine and projective structures on the torus
Complex projective structures and the Schwarzian derivative
Classification of complete affine 3-manifolds, Margulis spacetimes and generalizations

Anosov Representations

Olivier Guichard (Strasbourg)

Anosov representations are a special class of discrete subgroups of Lie groups that was pinned down by Franois Labourie in his study of the Hitchin component for $SL(n,\mathbb{R})$. In many aspects, Anosov representations can be viewed as the ”good” generalization of convex cocompact subgroups of rank one Lie groups. One nice feature of this class is the abundance of examples. As their name indicates, the definition of Anosov representations involves the hyperbolicity of a dynamical system. There are in turn other characterizations which express that the subgroup is ”very much” undistorted.

Metric projective geometry

Vladimir Matveev (Jena)

(Curved) projective structures, examples, metrisation equation in dimension 2, other projectively invariant equations, killing tensors with applications to possible number of solutions and to topology, how to solve overdetermined systems of PDE, degree of mobility, solution of a problem of Sophus Lie 1882, special feathures of projective structures in higher dimensions, conification constructions, Weyl theorem, number of solutions of metrisation equation, global ordering of eigenvalues and topology, Lie problems for higher dimensions and projective Lichnerowich conjecture, open problems and further directions of research.

Metric Möbius Geometry and Boundaries of neg. curved spaces

Viktor Schroeder (Zürich)

There is a well known and deep connection between the geometry of a hyperbolic space and the Möbius geometry of its ideal boundary at infinity. For example, the isometries of the classical hyperbolic space (in the upper half space model) are the extensions of the Möbius maps of its ideal boundary (which is the Euclidean space compactified with one point at infinity). This connection can be generalized to larger classes of negatively curved spaces, in particular to so called CAT(-1) spaces. It turns out that the boundary of a CAT(-1) space carries in a natural way a canonical Ptolemaic Möbius structure (we will introduce these concepts in detail). We will study Ptolemaic Möbius spaces in detail and will obtain cetain rigidity results. In particular, we will characterize boundaries of rank one symmetric spaces of noncompact type purely in terms of Möbius geometry.

Convex geometries of Teichmüller space: a comparative study

Sumio Yamada (Tokyo)

In this series of talks, we will demonstrate the viewpoint where the Teichmüller space is regarded as a convex body. Recall that the Teichmüller space is the space of conformal/complex structures defined on a topological surface. The space is known to be diffeomorphic to an open ball in $\mathbb{R}^n$. With the goal of understanding the moduli space of Riemann surfaces in mind, there are three important metrics defined on the space, namely the Teichmüller metric, the Weil-Peterson metric and the Thurston metric. We will compare the three geometries based upon the classical convex geometry of the Euclidean space.

Abstracts of Seminars

Thurston's asymmetric metric for unusual surfaces

Daniele Alessandrini

Thurston introduced its asymmetric metric for Teichmüller spaces of closed surfaces or for surfaces with punctures. This is a Finsler metric with very nice geometric properties and applications. Similar distances can be defined also for the Teichmüller spaces of surfaces with boundary and of surfaces of infinite topological type, even if these cases are much more delicate. I will discuss some properties of this distance in this more general setting.

Special Families of Hyperbolic Structures on Infinite Type Surfaces

Ozgur Evren

It is known that the topologies defined by the Teichmüller metric and the Length Spectrum metric may fail to coincide for surfaces of infinite type. Using a theorem which provides sufficient conditions for topological inequivalence of Teichmüller and Length Spectrum metrics in terms of length and twist parameters on a certain kind of an infinite type surface, we will construct an infinite parameter family of quasiconformally distinct hyperbolic structures with the property that the Length Spectrum metric and the Teichmüller metric define different topologies on the Teichmüller space of any one of the hyperbolic structures in the family.

Compactifications of locally symmetric spaces of infinite volume

Lizhen Ji

Given a semisimple Lie group G such as $SL(n, \mathbb{R})$, one question is what kinds of discrete subgroups Γ of G one should study and one can study effectively. This is closely related to the action of Γ on the symmetric space $X = G/K$, where K is a maximal compact subgroup, or the structure of the locally symmetric space $\Gamma \setminus X$.

When the symmetric space X is the real hyperbolic space (or more generally when X, or equivalently G, is of rank 1), there are several classes of discrete subgroups such as lattices, geometrically finite subgroups, convex cocompact subgroups. These groups and their associated hyperbolic spaces are well-studied.

But the situation with the higher rank case is much less clear. For example, the direct generalization of convex cocompact subgroup does not really work, and the notion of Anosov representations (or Anosov subgroup) by Labourie, Guichard & Wienhard, and related notions by Kapovich & Leeb & Porti, are right and appropriate generalizations. One important aspect of convex cocompact subgroups concerns compactifications of locally symmetric spaces (or geometry at infinity). We will discuss some questions, approaches and observations about compactifications of infinite volume locally symmetric spaces associated with Anosov subgroups.

Andreev's theorem on projective Coxeter polyhedra

Gye-Seon Lee

In 1970, E.M. Andreev gave a full description of 3-dimensional compact hyperbolic polyhedra with dihedral angles submultiples of pi. We call them hyperbolic Coxeter polyhedra. More precisely, given a combinatorial polyhedron C with assigned dihedral angles, Andreev's theorem provides necessary and sufficient conditions for the existence of a hyperbolic Coxeter polyhedron realizing C. Since hyperbolic geometry arises naturally as sub-geometry of real projective geometry, we can ask an analogous question for compact real projective Coxeter polyhedra. In this talk, I will give a partial answer to this question.
This is a joint work with Suhyoung Choi.

Teichmüller spaces as infinite polyhedra

Yukio Matsumoto

Let $T_{g,n}$ be Teichmüller space of type (g,n). Let $L : T_{g,n} \to \mathbb{R}$ be the piecewise analytic function which sends each point of $T_{g,n}$ to the minimum of the lengths of the non-trivial simple closed geodesics on the corresponding Riemann surface. Fix a sufficiently small positive number ε. Then the subset $P^{\varepsilon}_{g,n}$ defined by

$$\{p \in T_{g,n} | L(p) \geqq \varepsilon\}$$

is an "infinite polyhedron". By a simple observation using the complex of curves, the group of isometries of this polyhedron is proved to be isomorophic to the mapping class group of type (g,n). Also subgroups which preserve facets of $P^{\varepsilon}_{g,n}$ give a natural orbifold structure of the Deligne-Mumford compactification of the moduli space of type (g,n)

String kinematics and integrability in higher dimensions

Renzo Ricca

In this talk we review some fundamental results in geometric integrability for the time evolution of one-dimensional systems. Starting from the classical study of the intrinsic kinematics of a space curve, rooted in the original work of Da Rios (1906) [1, 2], we show how this is related to the dynamics of physical objects, such as vortex lines in an ideal fluid, that gives rise to the propagation of solitons. We show how intrinsic kinematics and integrable conditions can be both profitably extended to a higher dimensional context [3], where interesting relations between properties of minimal energy surfaces, integrable systems a nd string theory can be exploited.

[1] Da Rios, L.S. 1906 Sul moto dun liquido indefinito con un filetto vorticoso di forma qualunque. *Rend. Circ. Mat. Palermo* **22**, 117-135.

[2] Ricca, R.L. (1991) Rediscovery of Da Rios equations. *Nature* **352**, 561-562.

[3] Ricca, R.L. (1991) Intrinsic equations for the kinematics of a classical vortex string in higher dimensions. *Phys. Rev. A* **43**, 4281-4288.

Strictly convex functions on complete Finsler manifolds

Bankteshwar Tiwari

The existense of strictly convex functions on a complete Riemannian manifolds has influenced the topology of the manifold. In this talk, after a brief review of Riemann-Finsler Geometry, the properties of a complete Finsler manifold under the existense of strictly convex functions will be discussed.