Departments of Mathematics

NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

University of the Aegean

"Geometric Analysis", Karlovassi, Samos

31 May - 4 June 2010

Program

	Monday 31/05	Tuesday 1/06	Wed. 2	Thursday 3	Friday 4
9:00-10:00	9:30 Welcome	Knieper		Besson/Courtois	Tsolomitis
10:00-11:00	Papadopoulos	Besson/Courtois		Papadopoulos	Besson/Courtois
11:00-11:30	Coffee Break	Coffee Break		Coffee Break	Coffee Break
11:30-12:30	Knieper	Papadopoulos		Knieper	Students Talk
12:30-13:30	Besson/Courtois	Savo	Excursion	Tsapogas	Students Talk
13:30-17:30	Noon Break	Noon Break		Noon Break	
17:30-18:30	Balogh	Marias		Deroin	
18:30-19:30	Holopainen	El Soufi		Carron	

Excursion:

Departure at 10:00, destination Πυθαγόρειο (Pythagoreion) where we will visit the Eupalinian aqueduct (Ευπαλίνειο όρυγμα) , go to the sea and eat, then we will leave for Κοκκάρι (Kokkari) for a short stop and then we will go back to Καρλόβασι.

Conference Dinner:

On Thursday evening.

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Title and abstract of talk/courses

Three mini-courses

G. Besson/G. Courtois: Rigidity, entropy and volume;

G. Knieper: New results on harmonic manifolds and the Lichnerowicz conjecture;

A. Papadopoulos: Weak Finsler structures.

Z. Balogh:
Exceptional sets of absolute continuous curves for quasiconformal maps

A classical result in the theory of quasiconformal mappings says that any quasiconformal mapping is absolutely continuous on almost every line parallel to the coordinate axes. This result has been generalized to the setting of the Heisenberg group my Mostow and by Koranyi and Reimann. We shall investigate the size of the curve family for which this property fails.

G. Besson & G. Courtois:
Rigidity, entropy and volume

We shall present several constructions of maps between manifolds, or more general objects, due to Besson-Courtois and Gallot and show how this may be used to prove various rigidity results. Among them, we shall describe a simple proof of Mostow rigidity theorem for closed rank one symmetric spaces. This extends to a sharp inequality on the entropy of the geodesic flow on certain negatively curved closed manifolds as well as a computation of the minimal volume of closed hyperbolic manifolds. Extensions to the study of the structure of the fundamental group of certain closed hyperbolic manifolds shall be described together with the growth of groups acting on Hadamard manifolds.

S. Buckley:
Notions of nonpositive curvature in analysis

We discuss the relationships between CAT(0) and other notions of nonpositive curvature in a metric space. We also discuss some of the main applications in analysis of these conditions.

G. Carron:
L^2 harmonic forms on non compact manifold

These lectures aimed to give an overview of the bridge between topology and geometry provided by the properties of L^2 harmonic forms. During the first two lectures we will present the set of L^2 harmonic and its link with the L^2 cohomology. In the lectures 3 and 4, we will concentrate on the space of harmonic 1 form and its link with the number of ends; we will related certains answer to the question of recurrence and transcience of the Brwonian motion. Finally in the last two lectures, we will give several vanishing result for the space of L^2 harmonic forms and gives some toplogical consequnec of this vanishing result.

Prerequisits: Some Riemann geometry and De Rham Cohomology

B. Deroin:
Qualitative properties of differential equations in the complex domain

The lectures will focus on the study of singular holomorphic foliations on compact complex surfaces. An algebraic classification, in the spirit of Enriques classification of algebraic surfaces, has been achieved recently by McQuillen, Brunella and Mendes. However, many of these equations turn out to be of "general type", and we would like to understand their qualitative behaviour. For instance, a polynomial ODE in the complex plane of degree ≥ 2 defines a singular holomorphic foliation of general type on the projective plane. The lectures will cover several aspects: the geometry/topology of the leaves which typically are non compact Riemann surfaces, the study of the dynamics of the so-called holonomy pseudo-group, and the applications to the question of the ergodicity (with respect ot the Lebesgue measure), the structure of minimal sets etc.

I. Holopainen:
p-harmonic functions on negatively curved spaces

In 1979 Greene and Wu conjectured that a Cartan-Hadamard manifold M admits non-constant bounded harmonic functions if the sectional curvatures of M have an upper bound

$$K_M(P) \leq \frac{-C}{r^2(x)}$$

outside a compact set for some constant $C > 0$, where $r = d(\cdot, o)$ is the distance function to a fixed point $o \in M$ and P is any 2-dimensional subspace of $T_x M$. A Cartan-Hadamard manifold M can be compactified by adding a sphere at infinity (or a boundary at infinity), denoted by $M(\infty)$, so that the resulting space $\bar{M} = M \cup M(\infty)$ equipped with the cone topology will be homeomorphic to a closed Euclidean ball.

The conjecture of Greene and Wu is still open for dimensions $n \geq 3$. It can be approached by studying the so-called Dirichlet problem at infinity (or the asymptotic Dirichlet problem). Thus one asks whether every continuous function on $M(\infty)$ has a (unique) harmonic extension to M. In general, the answer is no since the simplest Cartan-Hadamard manifold $\mathbb{R}^n$ admits no positive harmonic functions other than constants. On the other hand, some kind of curvature lower bounds are needed even in the case of strictly negative sectional curvatures by counterexamples due to Ancona (and Borbély). The Dirichlet problem at infinity has been extensively studied during the last 30 years under various curvature assumptions.

In the talk I will survey studies on the Dirichlet problem at infinity for p-harmonic functions on Cartan Hadamard manifolds and on Gromov hyperbolic metric measure spaces. I will also describe the counterexample by Borbély and show that after a slight modification it applies to the case of p-harmonic functions as well. The talk is based on joint works with Urs Lang and Aleksi Vähäkangas.

G. Knieper:
New results on harmonic manifolds and the Lichnerowicz conjecture

Preliminary abstract:

The Lichnerowicz conjecture asserts that each complete harmonic Riemannian manifold is a symmetric space. In 1990 S. Szabo proved Lichnerowicz conjecture for compact simply connected spaces. However not much later in 1992 Damek and Ricci showed that in the non compact case the conjecture is wrong. In this talk we show that the Lichnerowicz conjecture for noncompact, simply connected harmonic manifolds X is true provided X has pure exponential volume growth and admits a compact quotient. Among them are all simply connected harmonic manifold of nonpositive curvature admitting a compact quotient. Previously this was only known in the case of negative curvature. We also remark that Damek- Ricci spaces have pure exponential volume growth.

M. Marias:
Singular Integrals on Hyperbolic manifolds

We present some recent results on the L^p-boundedness of the Riesz transform and spectral multipliers on kleinian groups.

A. Papadopoulos:
Weak Finsler structures

1. Convex sets;
2. Symmetrization;
3. Weak Finsler structures;
4. The tautological weak Finsler structure;
5. The Hilbert metric.

A. El Soufi:
The effect of the geometry on the eigenvalues of natural operators on manifold

The sequence of eigenvalues of the Dirichlet Laplacian on a bounded Euclidean domain satisfies several restrictive conditions such as : Faber-Krahn isoperimetric inequality, that is the principal eigenvalue is bounded above in terms of the volume of the domain, Payne-Pólya-Weinberger type universal inequalities, that is the k-th eigenvalue is controlled in terms of the $k-1$ previous ones, etc.

The situation changes completely as soon as Euclidean domains are replaced by compact manifolds. For example, according to results by Colin de Verdière and Lohkamp, given any compact manifold M of dimension $n \geq 3$, it is possible to prescribe arbitrarily and simultaneously, through the choice of a suitable Riemannian metric on M, a finite part of the spectrum of the Laplacian, the volume and the integral of the scalar curvature. Hence, Faber-Krahn and Payne-Pólya-Weinberger inequalities have no analog in this context.

In this short course, we will discuss the effect of the geometry on the eigenvalues. We will try to understand what kind of geometric situations lead to large eigenvalues for the Laplacian on manifolds of fixed volume, and what does such a Riemannian manifold look like once realized as a submanifold of a Euclidean space. On the other hand, we show that when the Laplacian is penalized by the squared norm of the mean curvature, then we obtain a Schrödinger type operator whose spectral behavior is very close to that of the Dirichlet Laplacian on Euclidean domains.

A. Savo:
Some aspects of the spectrum of the Laplacian on forms

We will focus on the eigenvalues of the Hodge Laplacian acting on differential forms on manifold of special type: namely, domains in Euclidean or Hyperbolic space and (minimal) hypersurfaces in a space form. The scope is to focus on the geometric aspect of the problem, trying to avoid some delicate technical points which often occur in the study of the Hodge Laplacian.

G. Tsapogas:
CAT(0) spaces and their dynamics

The subject of the presentation will be $CAT(0)$ spaces. A $CAT(0)$ space is a simply connected, complete, geodesic metric space X with curvature ≤ 0, where the notion of curvature is the local requirement for geodesic triangles to be thinner that the corresponding triangles in $\mathbb{R}^2$. Basic geometric properties of such spaces will be discussed, such as, uniqueness of geodesics, properties of asymptotic geodesics, description of the boundary ∂X of X via asymptotic geodesics, classification of isometries of X, actions of groups of isometries with special emphasis on the limit set of the action in ∂X.

The dynamics of the geodesic flow on a $CAT(0)$ space X with respect to discrete group Γ of isometries of X will then be studied. In particular, we will examine results and open questions for recurrence, transitivity and mixing of the geodesic flow on the quotient space X/Γ.

A. Tsolomitis:
The isotropic constant

We will present one of the most central problems of Asymptotic Geometric Analysis, known as the isotropic constant conjecture. Does there exist a universal constant $c > 0$ so that for every $n \in \mathbb{N}$ and for every convex body K in $\mathbb{R}^n$ of volume 1, there exists a subspace F of $\mathbb{R}^n$ of dimension $n-1$ so that the $n-1$ dimensional volume of $K \cap F$ is greater than c? We will present the problem and the best estimates so far (which still depend on n).

$$\Delta\varphi = -\nabla \cdot (\nabla\varphi)$$